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COMPARISON OF TWO SOLUTION STRATEGIES FOR USE WITH HIGHER-ORDER DISCRETIZATION SCHEMES IN FLUID FLOW SIMULATION

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SUMMARY

The paper describes and compares two different approaches to solving the equations of motion for fluid flow in a three-dimensional lid-driven cavity, when a higher-order approximation to convective transport is employed. One is based on the traditional pressure correction approach in conjunction with a pentadiagonal ADI solver; the other follows a new unsegregated variable and FAS multigrid methodology. The results generated by both approaches, for laminar flow conditions, at Reynolds numbers of 100 and 1000 are compared with each other and with corresponding solutions obtained with a well known low-order approximation to convection. Cross-reference is also made to flow in a two-dimensional cavity at the same Reynolds numbers.

KEY WORDS Unsegregated Multigrid High-order Finite-difference

1. INTRODUCTION

Less than 20 years ago it appeared to many in engineering circles that the numerical simulation of complex fluid flow phenomena offered limitless possibilities. However, it soon became apparent that finite difference approximations to convective transport suffered from one of two problems—either instability (central differencing) or numerical diffusion (upwind differencing). This apparently insoluble dichotomy led to the creation of the well known hybrid scheme,[1] which still enjoys widespread usage today. In the mid 1970s Raithby went some way towards addressing the problem of numerical diffusion and proposed the skew-upwind scheme,[2] but it was not until the end of the decade that a more satisfactory answer to the problem was found by Leonard in the form of quadratic upwind interpolation.[3] It is now generally accepted that the latter represents the best approximation to convective transport for problems in which profiles of the dependent variables vary relatively smoothly.[4,5] It would be naive to pretend that quadratic upwind interpolation represented a cure for all ills; indeed, it has its own shortcomings, namely a characteristic lack of boundedness. Fortunately, this does not appear to present a problem for the test case considered here and the reader is referred elsewhere[4] for a more detailed description of and possible answer to this rather undesirable feature of higher-order schemes.

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During the above period, the SIMPLE algorithm,[6] and lately its variants,[7] has emerged as arguably the most successful solution algorithm for simulating fluid flow problems. The continuum equations governing such flows are solved numerically using a segregated or pressure correction approach. This, together with a tridiagonal matrix algorithm (TDMA) employing an alternating direction implicit (ADI) procedure, has resulted in an extremely efficient solution strategy that has reigned more or less supreme. However, an obvious drawback to this methodology is that it weakens the coupling between velocity and pressure and can impede progression towards a converged solution.[8] The cost of obtaining solutions depends strongly on how well the solution procedure treats the inter-equation coupling. In recent studies of Gaskell and Wright[9,10] it has been shown that implementation of Vanka's[11] symmetric coupled Gauss–Seidel (SCGS) technique represents a first step in dealing with this problem. The momentum and continuity equations are retained in their original form—a pressure correction procedure is not required and the coupling between the equations is preserved. Similarly, the recent rapid advance in state-of-the-art computational fluid dynamics, in the form of multigrid methods acting as convergence accelerators, is now beginning to challenge the superiority of the traditional solver.[9,11]

Section 2 describes the test problem and governing equations of motion. This is followed by a detailed description of the numerical procedures adopted, in particular the SCGS smoothing technique and the multigrid algorithm employed, in Section 3. The results are compared in Section 4 and conclusions drawn in Section 5.

2. TEST PROBLEM AND GOVERNING EQUATIONS

The three-dimensional lid-driven cavity, for laminar flow conditions, represents an ideal test problem for evaluating the relative merits of the solution strategies outlined below. In the cubic cavity shown in Figure 1 the velocities are equal to zero on all faces except in the xz plane for $y = 0$,

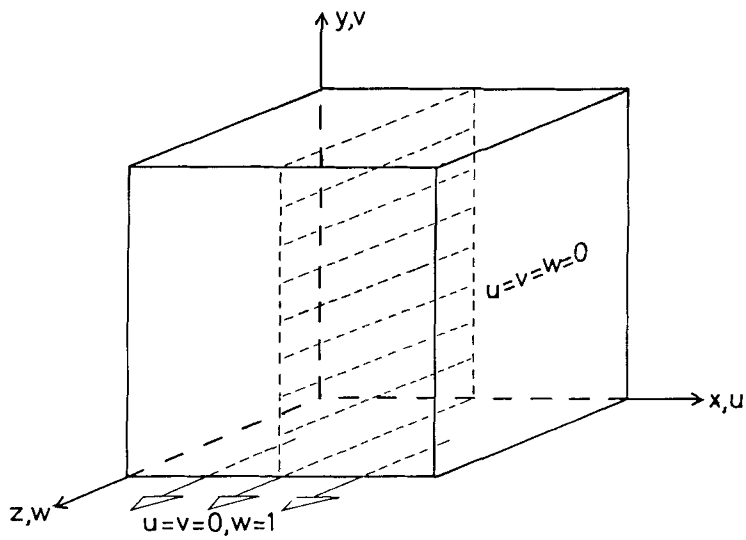

Figure 1. Three-dimensional lid-driven cavity problem with the plane of symmetry shown

where $w=1$. The equations governing the flow, written in non-dimensional form, are

$$\frac{\partial u^2}{\partial x}+\frac{\partial uv}{\partial y}+\frac{\partial uw}{\partial z}=-\frac{\partial p}{\partial x}+\frac{1}{Re}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right), \quad (1)$$

$$\frac{\partial uv}{\partial x}+\frac{\partial v^2}{\partial y}+\frac{\partial vw}{\partial z}=-\frac{\partial p}{\partial y}+\frac{1}{Re}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right), \quad (2)$$

$$\frac{\partial uw}{\partial x}+\frac{\partial vw}{\partial y}+\frac{\partial w^2}{\partial z}=-\frac{\partial p}{\partial z}+\frac{1}{Re}\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right), \quad (3)$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0. \quad (4)$$

A primitive variable (u, v, w, p) and control volume formulation has been chosen in line with current engineering practice, although in terms of multigridding it is not necessarily the most desirable option for the chosen test problem; also the use of a staggered grid arrangement[12] complicates multigrid interpolation procedures.

3. NUMERICAL PROCEDURES

3.1. Discretization

The key to the accurate simulation of convection-dominated flows depends upon the method adopted to discretize the governing equations of motion, in particular the troublesome non-linear convective terms. The characteristic lack of boundedness associated with high-order approximations to convection has meant that the hybrid scheme[1] has, on the whole, remained preferable for the elucidation of problems which contain steep gradients in one or more of the dependent variables. The desire of computational fluid dynamicists to predict evermore complex turbulent flows has more-or-less sealed their fate in this respect. Fortunately, the test problem considered here does not feature sharp changes in gradient for the velocity field, thus allowing the use of a higher-order approximation to convection; namely, Leonard's quadratic upward interpolation for convection kinematices (QUICK). The debate surrounding the choice of a suitable approximation to convective transport is far reaching and fraught with controversy. The theme of this paper precludes us from expanding further; however, the reader's attention is drawn to a recent publication by Gaskell and Lau[4] in this area.

The diffusion terms contained in the governing equations are approximated using central differencing in line with standard practice.

3.2. A pressure correction/PDMA formulation

Solution of the pressure and velocity fields. The SIMPLE algorithm, attributed to Patankar and Spalding,[6] represents a remarkably successful implicit solution procedure for determining the coupled velocity and pressure fields associated with fluid flow problems. The method itself is extremely well documented and as such it is not reviewed here; rather the reader is referred to the relevant text.[13] There is, however, one important feature of our approach that differs from that of Patankar and Spalding, namely that the pressure correction is over- rather than under-relaxed. This has the desired effect of rendering pressure correction values, between iterative cycles, smaller. A more detailed explanation of the reasons behind this modification to the existing algorithm can be found elsewhere.[14] It is sufficient to state here that in common with accepted practice the following steps are taken when applying the SIMPLE algorithm:

(i) A pressure field is assumed *a priori.*
(ii) This pressure field is then used to give the corresponding approximate velocity field.
(iii) The velocity and pressure fields are corrected if the former does not satisfy the continuity equation.
(iv) Steps (ii) and (iii) are repeated until a converged solution is obtained.

Solution of coefficient matrices. When an ADI procedure is adopted, higher-order finite difference approximations to convective transport often result in a coefficient matrix with a bandwidth of five in the chosen co-ordinate direction, unlike the hybrid approximation which generates a matrix with a bandwidth of three. The latter is ideally suited for use with a TDMA. Although a pentadiagonal matrix algorithm (PDMA) represents the obvious choice of solver in the former case, this has not proved popular. It has become common practice[5,15] to recast the QUICK scheme, for example, in such a way that a TDMA can be used. There are of course disadvantages to following such a procedure:

(i) The finite difference equations are not solved 'exactly' on the current line–nodal points are dumped to the source.
(ii) The resultant scheme is invariably more complex following reformulation; programming effort is increased.
(iii) Reformulation of schemes such as QUICK and the dumping of terms into the source must contribute to a decrease in the overall stability of the coefficient matrix system.

There are, however, some authors who have opted to use a PDMA in conjunction with higher-order discretization schemes, Agarwal,[16] Galpin *et al.*[17] and Seyed *et al.*[18] for example, the latter having expressed their doubts about the robustness of the PDMA as a solver. It is arguable that these doubts may be premature, since no definitive evidence is given for this assertion. Some of the discretization schemes examined therein are of sufficient complexity to mask any desirable features of the solver.

The PDMA has the combined advantage of simplicity and a low storage requirement. This solver requires no special reformulation of higher-order discretization schemes or the need to dump terms into the source.

If the (j, k)th values of a dependent variable ϕ along an ith grid line are fixed at their previous iterative value, then the set of quasi-linear algebraic equations thus generated can be written as

$$-A_{i-2}\phi_{i-2}-A_{i-1}\phi_{i-1}+A_i\phi_i-A_{i+1}\phi_{i+1}-A_{i+2}\phi_{i+2}=S_i. \tag{5}$$

The coefficients in A contain both convective and diffusive fluxes, and S_i is a source term such that

$$S_i=\sum A_m\phi_m+S, \quad m=j\pm 1, j\pm 2, k\pm 1, k\pm 2.$$

Applying (5) at all nodal points ζ along this grid line results in a pentadiagonal coefficient matrix which can be easily factorized by LU-decomposition to generate the following recursive procedure:

$$(\phi_i)_\zeta=v_\zeta-\delta_\zeta(\phi_i)_{\zeta+1}-\lambda_\zeta(\phi_i)_{\zeta+2}, \tag{6}$$

where

$$v_\zeta=((S_i)_\zeta-\beta_\zeta v_{\zeta-1}-\sigma_\zeta v_{\zeta-2})/\gamma_\zeta \tag{7}$$

with

$$
\begin{aligned}
\sigma_\zeta &= -(A_{i+2})_\zeta,\\
\beta_\zeta &= -(A_{i+1})_\zeta - \sigma_\zeta\delta_{\zeta-2},\\
\gamma_\zeta &= (A_i)_\zeta - \sigma_\zeta\lambda_{\zeta-2} - \beta_\zeta\delta_{\zeta-1},\\
\delta_\zeta &= -((A_{i-1})_\zeta + \beta_\zeta\lambda_{\zeta-1})/\gamma_\zeta,\\
\lambda_\zeta &= -(A_{i-2})_\zeta/\gamma_\zeta.
\end{aligned}
$$

For a solution over the domain of interest, the above procedure is applied to each of the i grid lines in turn, the most recently calculated values on the previous grid line being used. After one sweep the procedure is applied along the j and k grid lines. This ADI procedure can be repeated any number of times to achieve a desired level of accuracy for the solution of one dependent variable.

3.3. An unsegregated solution/FAS multigrid formulation

The smoothing technique—symmetrical coupled Gauss–Seidel. If one's ultimate goal is to enhance convergence with a multigrid approach, it is important to realize from the start that any benefits to be derived are dependent on the smoothing technique employed. If an inefficient technique is used, the advantages of multigrids can be completely annulled. Attempts have been made to adapt pressure correction techniques to multigrids, but there are problems associated with this.[19, 20] Similarly, other authors have implemented a multigrid technique for either the pressure equation or the velocity equations. The symmetrical coupled Gauss–Seidel technique[11] used here solves for u, v, w and p simultaneously, thereby maintaining the coupling of the equations. It is simple and offers the advantage of a low operation count and minimal storage requirement. As a first step to assessing the use of an unsegregated solver with a multigrid technique, it is easily implemented and reasonably efficient.

For each control volume six velocities and one value of the pressure are updated simultaneously by inverting a 7 × 7 matrix

$$
\begin{bmatrix}
A^u_{ijk} & & & & & & -1/h\\
& A^u_{i-1jk} & & & & & 1/h\\
& & A^v_{ijk} & & & & -1/h\\
& & & A^v_{ij-1k} & & & 1/h\\
& & & & A^w_{ijk} & & -1/h\\
& & & & & A^w_{ijk-1} & 1/h\\
1/h & -1/h & 1/h & -1/h & 1/h & -1/h & 0
\end{bmatrix}
\begin{bmatrix}
u'_{ijk}\\ u'_{i-1jk}\\ v'_{ijk}\\ v'_{ij-1k}\\ w'_{ijk}\\ w'_{ijk-1}\\ p'_{ijk}
\end{bmatrix}
=
\begin{bmatrix}
r^u_{ijk}\\ r^u_{i-1jk}\\ r^v_{ijk}\\ r^v_{ij-1k}\\ r^w_{ijk}\\ r^v_{ijk-1}\\ r^c_{ijk}
\end{bmatrix}
$$

This matrix is doubly bordered, diagonal and sparse, and can be solved 'exactly' using a form of LU-decomposition. Relaxation of the updates is required owing to the non-linearity of the algebraic equations and the necessary use of old values for u, v, w and p when evaluating the matrix coefficients and the residuals. The velocity updates are multiplied by a factor α_1 and the pressure updates by α_2 once they have been calculated. The choice of α_1 and α_2 is not absolutely critical, but it does depend upon the value of the Reynolds number. QUICK is seen to exhibit a greater sensitivity to the value of these relaxation factors (in particular α_1) than is the case with hybrid discretization.[9] This set of equations is solved for each control volume, first in the direction of increasing i, then j and then k. As a consequence of this each velocity is updated twice. Vanka observed that this ensured the stability which a single update method lacked.

Solutions obtained by considering alternative cells only, in two separate sweeps, were also investigated, but this approach was found to offer no obvious advantages, particularly with the QUICK discretization which generates a large computational molecule.

The multigrid scheme. The central philosophy behind multigridding is that errors of wavelength λ are most easily eliminated on a mesh of size h when $\lambda \simeq h$. Once this error has been smoothed, on a given grid the convergence rate decreases rapidly. In view of this a hierarchy of grids of different mesh sizes is used to solve the fine grid problem (see Figure 2). A representation of this problem is set up on coarser grids and these are used to calculate corrections to the fine grid solution. With multigridding one has a very fast solver for systems of algebraic equations, since relaxation on each level is extremely efficient. Each grid is very effective at eradicating a particular wavelength component of the error. The efficiency of the method should be independent of the shape of the solution domain, the form of the boundary conditions and the smoothness of the solution. Within this overall concept there are several different strategies that can be adopted.

The multigrid strategy. The problem considered here is highly non-linear and can be solved in one of two ways:

(a) by performing a global linearization using a Newton method followed by the application of a linear multigrid, or
(b) by employing a full approximation storage (FAS) algorithm.

The latter approach is preferable and was therefore adopted here. Further details of the non-linear multigrid approach are readily accessible elsewhere.[9, 21]

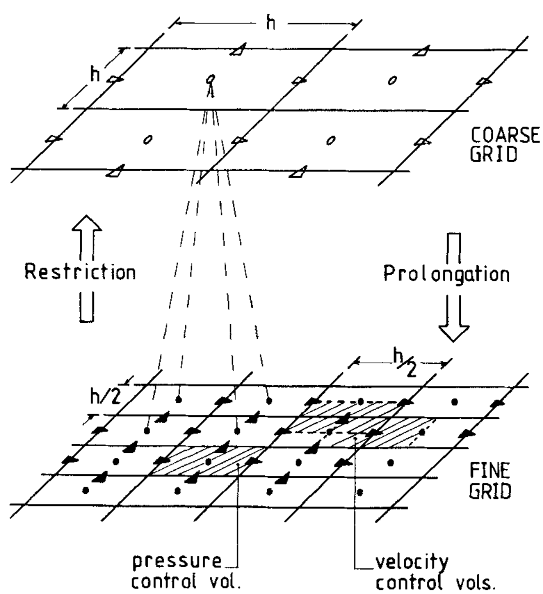

Figure 2. Multigrid nodal configuration showing coarse to fine (prolongation) and fine to coarse (restriction) data transfer between grids: o, coarse grid scalar; •, fine grid scalar; ◁, coarse grid velocity, ◀, fine grid velocity

Several strategies have been proposed and implemented for cycling between grids in order to smooth errors efficiently.[21] However, the one employed here is geared to home in on the grid with the largest residual, starting the smoothing cycle there. The reasoning behind this approach is that this grid will be the most efficient to work on in terms of smoothing errors. It has been applied very effectively to a number of problems by Falle and Wilson.[22] Therefore in performing one FAS multigrid cycle the following steps are taken:

(i) The equations are set up on all grids.
(ii) The residual of the solution of the restricted problem on each grid is calculated. The level with the highest residual is selected for smoothing.
(iii) The solution is smoothed on this grid until the error has been reduced by a factor γ. This factor can be varied to give a fast rate of convergence. In the present work γ is set to 0·5 on the current finest grid and 0·1 elsewhere. The final and initial solutions are used to calculate a correction.
(iv) This correction is prolonged onto the next finer grid and added to the current solution there. This solution is smoothed until the error has been reduced by a factor γ.
(v) Steps (iii) and (iv) are repeated until the finest grid has been corrected and smoothed.

Interpolation, boundary conditions and convergence criterion. In all cases information is transferred between grids using linear or bilinear interpolation. When interpolating near boundaries, no boundary values are used; instead a zero derivative is imposed. In order to improve the accuracy of the correction in this situation, cells adjacent to the boundary are updated after the correction has been applied and before the first current fine grid iteration. It has been observed that this measure can reduce CPU time by up to 30%.

Unlike Vanka we have made use of the symmetry property of the problem; solving for one half of the cavity and applying a zero-derivative boundary condition for v and w, and a zero-value condition for u at the symmetry plane shown in Figure 1. Care must be taken when solving with von Neumann boundary conditions. In the present work this condition is only applied on the current fine mesh, as this is the only place where a solution of the full partial differential equation is sought. The problem is solved on coarser grids by imposing a Dirichlet boundary condition, defined in terms of the current fine grid and its associated residuals. In the present study the first-order upwind approximation is applied at the boundaries of the solution domain and the computational grids quoted in the text refer to internal nodes only.

The results presented in the next section are for converged solutions where the residual norm

$$\| r \| = \frac{\left(\sum_{i,j,k} ((r^u_{ijk})^2 + (r^v_{ijk})^2 + (r^w_{ijk})^2 + (r^c_{ijk})^2) \right)^{1/2}}{4 \times iN \times jN \times kN} \tag{8}$$

is less than 10^{-5}.

4. RESULTS

Before proceeding to analyse the results for the cubic cavity, it is worthwhile collapsing the problem to a computationally less expensive scenario, namely a two-dimensional flow situation ($x = u = 0$ in Figure 1). Table I gives a comparison between the various solution strategies, for flows at Reynolds numbers of 100 and 1000, obtained with different mesh densities. The first thing

Table I. Solution times (in CPU seconds on an Amdahl 5860) for a square cavity obtained using SCGS/non-multigrid (SCGS), SCGS/multigrid (SCGSM) and SIMPLE/PDMA (SP) solution strategies when the QUICK scheme is employed. Results obtained for a multigrid version of the hybrid scheme are included as a comparison. Bracketed terms represent projected values based on equation (9)

$Re = 100$ Mesh	SCGS	SCGSM		SP
$iN \times jN$	QUICK	Hybrid	QUICK	QUICK
4 × 4	0·05	0·06	0·07	—
8 × 8	0·41	0·34	0·39	0·36
16 × 16	4·59	1·54	1·69	3·12
32 × 32	59·06	6·46	7·41	28.20
64 × 64	641·09	22·24	22·53	366·00
128 × 128	7007·57	69·51	69·67	3142·80
256 × 256	(76500·00)	311·47	244·47	(27000·00)

$Re = 1000$ Mesh	SCGS	SCGSM		SP
$iN \times jN$	QUICK	Hybrid	QUICK	QUICK
4 × 4	0·07	0·05	0·13	—
8 × 8	1·49	0·35	1·27	0·92
16 × 16	16·44	2·37	7·78	3·40
32 × 32	87·27	14·56	35·80	19·18
64 × 64	829·25	90·39	149·97	267·00
128 × 128	8559·77	358·23	546·43	2371·20
256 × 256	(88000·00)	1127·27	2124·24	(21000·00)

to notice is that the SCGSM solution strategy results in a relationship between CPU time and number of grid points, N, of the form

$$\text{CPU} \propto N^{\beta}, \tag{9}$$

where $\beta \simeq 1$. This is what one would expect to attain with a multigrid method. Secondly, SCGS without multigridding is no match for the traditional SIMPLE with PDMA (SP) solution strategy, at all mesh densities. Similarly, at low mesh densities, particularly for $Re = 1000$, SP out performs SCGSM, but this trend is dramatically reversed as N increases. For example, at $Re = 100$ a solution is found on a 64 × 64 grid using SCGSM in less time that it takes to find the solution on a 32 × 32 grid with an SP approach.

It would appear from the evidence presented in this table that as the Reynolds number is increased, the superiority of the SCGSM over the SP solution strategy only becomes apparent at higher mesh densities. This is only a speculative assertion, and flow at higher Reynolds number will need to be examined before any definite conclusion can be drawn. Replacing the point-by-point SCGS solver with a line-by-line technique should improve this trend still further. Similarly, the use of quadratic interpolation for the transfer of information between grids may enhance the performance of the multigrid technique. Figure 3 shows plots of the vorticity and streamfunction

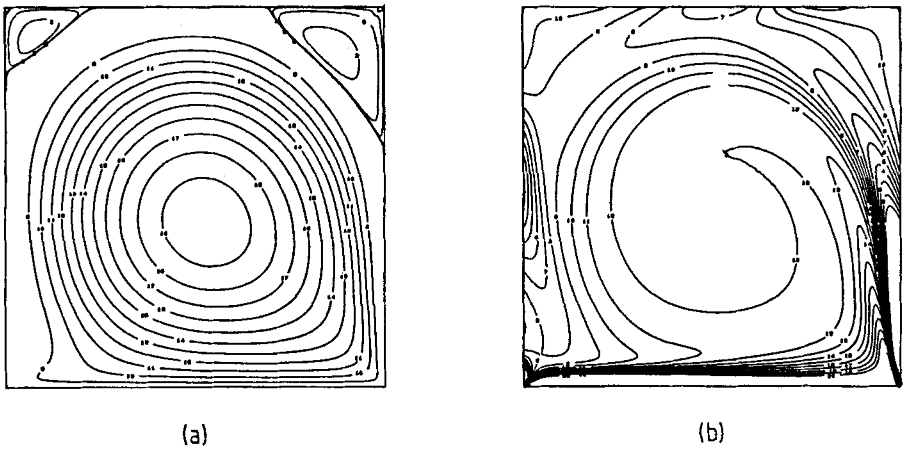

Figure 3. (a) Streamfunction and (b) vorticity contours for flow in a two-dimensional lid-driven cavity at Reynolds number 1000, obtained on a 256 × 256 mesh with QUICK

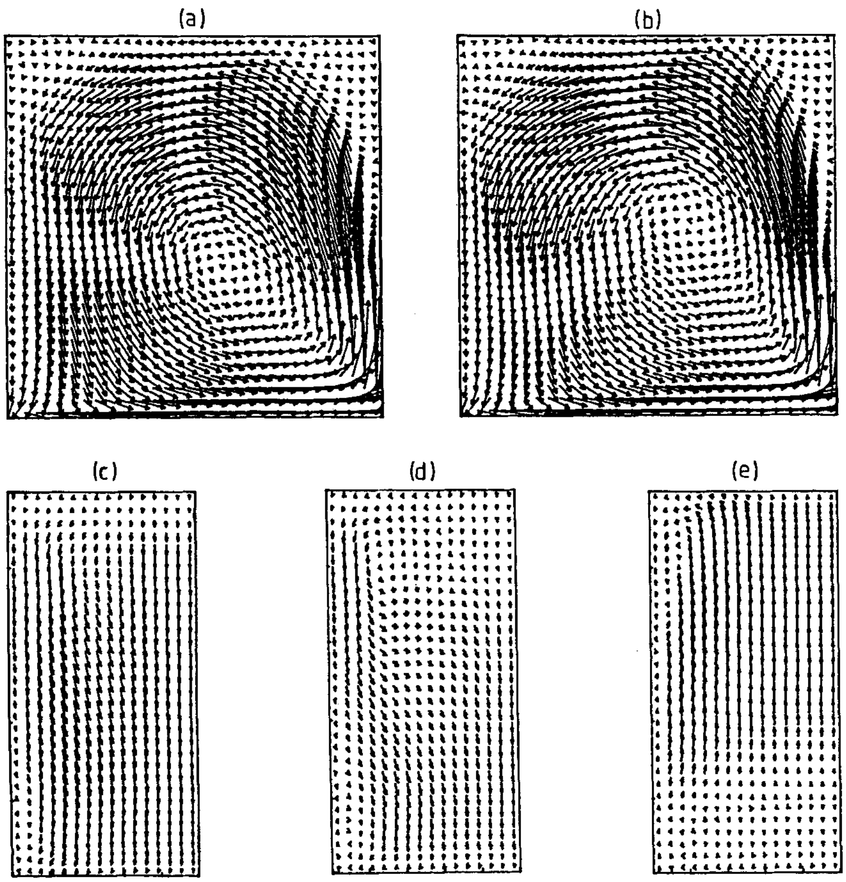

Figure 4. Velocity vectors obtained with hybrid for Reynolds number 1000: (a) $x=0.125$; (b) $x=0.5$; (c) $z=0.375$; (d) $z=0.5$; (e) $z=0.75$

at a Reynolds number of 1000 obtained on a 256 × 256 grid. These results are in excellent agreement with the predictions of other authors; see, for example, Ghia *et al.*[23]

Focusing now on the three-dimensional cavity, it can be seen that solutions are obtained for a large number of grid points in reasonably fast times. The main features of the flow are clearly resolved on a 32 × 32 × 16 mesh and at a Reynolds number of 1000 are in good agreement with those of other authors[11] (Figures 4 and 5). However, as one would anticipate, the hybrid solution is much more diffuse, failing to adequately capture the corner eddy at $z=0{\cdot}5$ (Figure 4(d)). In Figure 6 the velocity predictions for v and w at the central plane on $x=0{\cdot}5$ are presented. It can be seen quite clearly that although the results obtained with both schemes at a Reynolds number of 100 are comparable, the hybrid prediction differs considerably from that given by the QUICK scheme at a Reynolds number of 1000—highlighting once again the diffuse nature of the former.

In Table II a comparison is made between the SP and SCGSM solution strategies for both the QUICK and hybrid schemes, again for flow at Reynolds numbers of 100 and 1000. Once again it is evident that considerable (although less dramatic) savings in CPU time are achieved using the SCGSM solution strategy. Notice that for the three-dimensional computations there is no switch over in benefit of use between the SP and SCGSM approaches; that is, the latter is always faster.

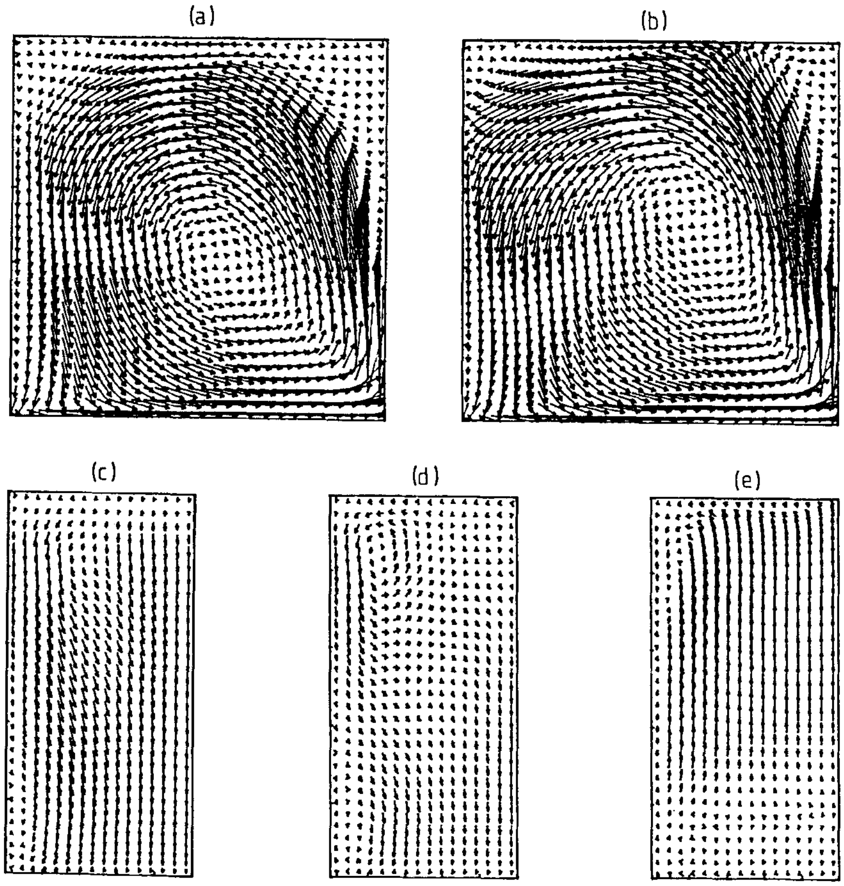

Figure 5. Velocity vectors obtained with QUICK for Reynolds number 1000: (a) $x=0{\cdot}125$; (b) $x=0{\cdot}5$; (c) $z=0{\cdot}375$; (d) $z=0{\cdot}5$; (e) $z=0{\cdot}75$

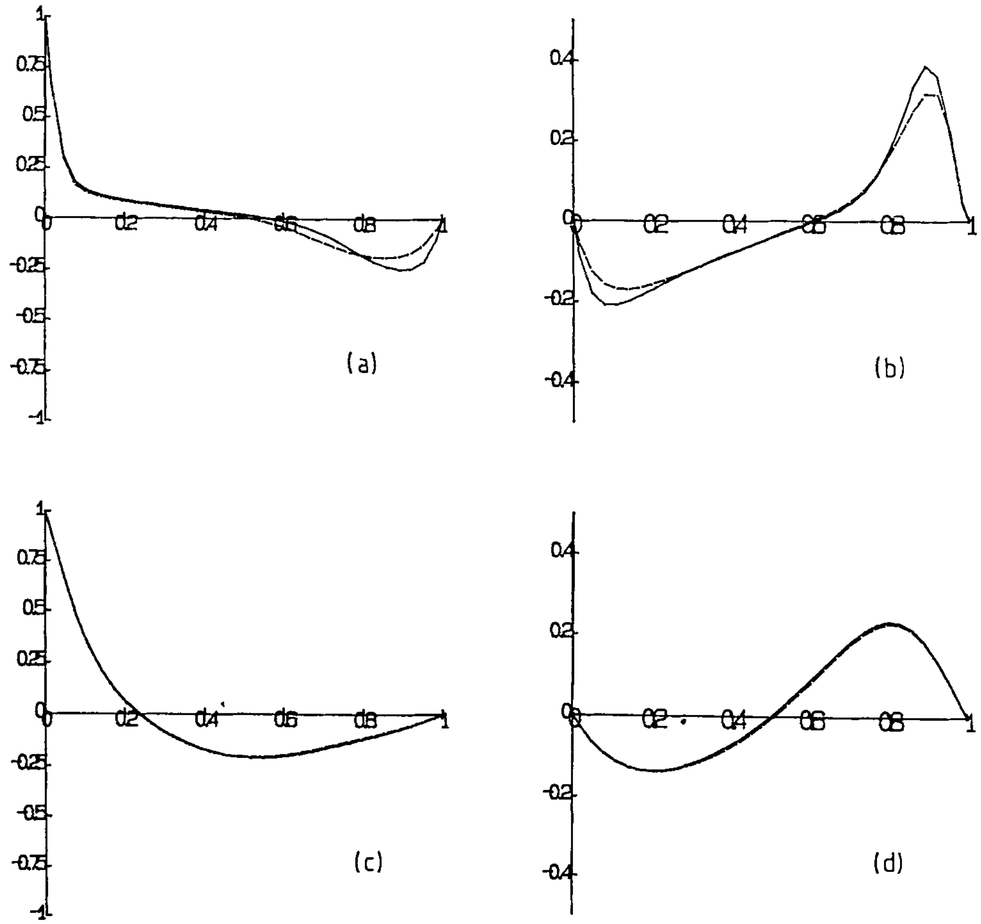

Figure 6. Velocity profiles at the central plane ($x=0{\cdot}5$) for QUICK (full curves) and for hybrid (broken curves): (a) $Re=1000$, w on the line $z=0{\cdot}5$; (b) $Re=1000$, v on the line $y=0{\cdot}5$; (c) $Re=100$, w on the line $z=0{\cdot}5$; (d) $Re=100$, v on the line $y=0{\cdot}5$

5. CONCLUDING REMARKS

Both of the solution strategies examined give accurate predictions for the flow under consideration when used in conjunction with the QUICK discretization scheme. No difficulties were experienced with using a PDMA to obtain solutions in two and three dimensions using the SIMPLE approach. The present study therefore lends support to and recommends the use of a PDMA to solve for the coefficient matrices which occur as a result of employing higher-order approximations to convection for the solution of recirculating fluid flow problems.

The FAS multigrid/SCGS approach results in economically more viable solutions, particularly when a fine mesh is used. It is believed that even greater improvements in CPU time can be achieved by replacing SCGS with a different smoothing technique and by making use of quadratic rather than linear interpolation when transforming information between grid levels. Both these options are currently under investigation.

Table II. Solution times (in CPU seconds on an Amdahl 5860) for a cubic cavity obtained using the SCGSM and SP solution strategies in conjunction with the hybrid and QUICK discretization schemes. Bracketed terms represent projected values based on equation (9)

$Re=100$	SCGSM		SP	
Mesh N	Hybrid	QUICK	Hybrid	QUICK
4 × 4 × 2	0·17	0·17	—	—
8 × 8 × 4	1·37	1·72	—	—
16 × 16 × 8	10·92	14·02	64·00	94·10
32 × 32 × 16	88·31	86·56	1061·00	1630·00
64 × 64 × 32	659·18	610·18	(17600·00)	(28000·00)

$Re=1000$	SCGSM		SP	
Mesh N	Hybrid	QUICK	Hybrid	QUICK
4 × 4 × 2	0·17	0·17	—	—
8 × 8 × 4	1·77	8.79	—	—
16 × 16 × 8	17·19	60·86	44·31	75·20
32 × 32 × 16	212·75	446·93	943·12	1430·01
64 × 64 × 32	3350·81	3383·26	(20100·00)	(27200·00)

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APPENDIX: NOMENCLATURE

A	coefficient of solution matrix
CPU	central processing unit
iN, jN, kN	number of nodes in x, y, z directions respectively
N	total number of nodes
p	pressure
r^u	residual in the u-equation
r^v	residual in the v-equation
r^w	residual in the w-equation
r^c	residual in the continuity equation
Re	Reynolds number
S	source term
S_i	source term at node i
u, v, w	velocity components in x, y, z directions respectively
x, y, z	Cartesian co-ordinates
ϕ	dependent variables
$\|\cdot\|$	second norm

Superscripts

u, v, etc.	denotes belonging to
$'$	denotes updates

Subscripts

$i, j, k, i-1, j-1, k-1$, etc. denote values at nodal points

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